Popper's test of Quantum Mechanics

Albert Bramon[1] and Rafel Escribano[2]

[1] Grup de Física Teòrica, Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain bramon@ifae.es
[2] Grup de Física Teòrica and IFAE, Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain Rafel.Escribano@ifae.es

A test of quantum mechanics proposed by K. Popper and dealing with two-particle entangled states emitted from a fixed source has been criticized by several authors. Some of them claim that the test becomes inconclusive once all the quantum aspects of the source are considered. Moreover, another criticism states that the predictions attributed to quantum mechanics in Popper's analysis are untenable. We reconsider these criticisms and show that, to a large extend, the 'falsifiability' potential of the test remains unaffected.

1 Introduction

Heisenberg's principle is the key feature of quantum mechanics [1] and plays the central role in many relevant discussions concerning counterintuitive quantum phenomena. This is particularly true when applied to composite quantum systems consisting of two-particle entangled states. A well known case is the Einstein-Podolsky-Rosen paradox which appeared in 1935 short after a preliminary and lesser known proposal by Karl Popper [2]. The latter "Popper's test" of quantum mechanics has been subsequently reformulated and improved by Popper himself [2]–[5], and reconsidered by several authors [6]–[12]. A look at the most recent of these papers shows that the controversy is still open. In the present note we attempt to clarify some points of such a basic issue.

Following Popper [5], we consider a source S decaying symmetrically into pairs of photons or equal-mass particles. In their center-of-mass frame, the two particles of each pair are assumed to be simultaneously emitted from the origin and to travel in opposite directions. Assume, for concreteness, that their trajectories are contained in the plane defined by the horizontal x-axis and the vertical y-axis. Two vertical screens are symmetrically placed at equal distances d, left (L) and right (R) of the source, thus intersecting perpendicularly the horizontal axis at $x = \pm d$. The left and right screens have s_L- and s_R-wide slits centered around the horizontal axis at $x = -d$ and $x = +d$, respectively. Some pairs of the emitted particles will then pass through the slits and will

trigger coincidence detectors placed far away[3] and completely covering the vertical space behind each slit. The vertical y-component of the momentum of the left- and right-moving particles, $(k_1)_y \equiv k_1$ and $(k_2)_y \equiv k_2$, can thus be measured with this experimental setup. The other, non-vertical components of $\mathbf{k}_1$ and $\mathbf{k}_2$ play a secondary role and the discussion thus centers on how the observed distributions for k_1 and k_2 depend on the slit widths, s_L and s_R. Assume, for instance, that one measures the k_1 and k_2 distributions for particle pairs with both members passing through equal-width slits, $s_L = s_R = 2a$. Since this amounts to position measurements with $\Delta y_{1,2} \simeq a$, from diffraction theory or Heisenberg's principle one expects $\Delta k_{1,2} \simeq 1/2a$ for the dispersion of the vertical momenta behind each slit. According to Popper's proposal, by two suitable modifications of the previous setup one can experimentally discriminate between his own (propensity) interpretation of quantum mechanics and Copenhagen's orthodoxy [5].

A first possibility —case (i)— consists in considering what happens to Δk_2 when one runs the experiment with the same $s_L = 2a$ as before but with a much wider s_R, $s_R \gg 2a$. Note that one still performs a position measurement with $\Delta y_1 \simeq a$ on the particle passing through the left-side slit and that, because of the entanglement, the right-moving particle has then to be in a state with $\Delta y_2 \simeq a$ when it passes through the vertical plane at $x = +d$. According to Popper [5], quantum mechanics predicts the same $\Delta k_2 \simeq 1/2a$ as before, whereas his own (propensity) approach predicts the disappearance of such a dispersion in Δk_2. Moreover —case (ii)— one can then narrow the left-slit width s_L while maintaining the same wide and fixed $s_R \gg 2a$, as before. According to quantum mechanics this narrowing of s_L implies larger values for $\Delta k_2 \simeq 1/2a$, while the opposite is expected from Popper's approach [5].

The proposal summarized in the previous two paragraphs —a crucial experiment, according to Popper— has however been criticized by several authors. All these criticisms are based on the fact that the source S cannot be exactly localized at the origin and perfectly at rest, as required to argue that the two entangled final particles separate from the origin strictly with opposite momenta. The undecayed source itself or, better, the global two-particle final system has to obey Heisenberg's principle and, accordingly, the vertical components of the CM-position, $(y_1 + y_2)/2$, and total momentum, $k_1 + k_2$, must satisfy $\Delta(k_1 + k_2)\Delta\left[(y_1 + y_2)/2\right] \geq 1/2$. Once this constraint is imposed, the analyses of Refs. [6, 7] —based on simple and intuitive geometrical arguments— claim that Popper's proposal is no longer able to discriminate between the two approaches. Similarly, the discussion in [11, 12] —based on a simplified wave function for the two-particle system which obeys Heisenberg's principle— claims that for case (ii) standard quantum mechanics predicts no

[3] These detectors are distributed in slightly different configurations according to the various authors. If the distance to the slits is large enough, all the various distributions become equivalent.

increase of Δk_2 when narrowing the left-side slit. A claim which contradicts Popper's original analysis [5] and would make his test inconclusive as well. We now proceed to discuss that these claims —based on simple geometrical arguments and a too naive wave function— are not completely justified and that Popper's test maintains most of its valuable 'falsifiability' potential.

2 Entangled two-particle state

Consider the following wave function describing the behavior of an entangled two-particle system

$$\Psi(y_1, y_2; t) = \int\int dk_1 dk_2 \Psi(k_1, k_2; t) \frac{e^{ik_1 y_1}}{\sqrt{2\pi}} \frac{e^{ik_2 y_2}}{\sqrt{2\pi}} \ , \tag{1}$$

where

$$\begin{aligned} \Psi(k_1, k_2; t) &= \frac{1}{\sqrt{\pi\sigma_+\sigma_-}} e^{-\frac{1}{4\sigma_+(t)^2}(k_1+k_2)^2} e^{-\frac{1}{4\sigma_-(t)^2}(k_1-k_2)^2} \\ &= \frac{1}{\sqrt{\pi\sigma_+\sigma_-}} e^{-\frac{1}{4}\left(\frac{1}{\sigma_+(t)^2}+\frac{1}{\sigma_-(t)^2}\right)(k_1^2+k_2^2)} e^{-\frac{1}{4}\left(\frac{1}{\sigma_+(t)^2}-\frac{1}{\sigma_-(t)^2}\right)2k_1k_2} \ , \end{aligned} \tag{2}$$

$\frac{1}{\sigma_\pm(t)^2} \equiv \frac{1}{\sigma_\pm^2} + i\frac{t}{m}$ accounts for the time evolution along the relevant, vertical y-axis, and m is the mass of each particle. Here and in what follows, integrations extend from $-\infty$ to $+\infty$ unless otherwise is stated.

Note that for the global system we have chosen a Gaussian wave packet [1] with $\Delta(k_1 + k_2) = \sigma_+$. This allows for analytical computations and at the decay time, $t = 0$, one has $\Delta(k_1 + k_2)\Delta\left[(y_1 + y_2)/2\right] = 1/2$, which is the minimum value compatible with Heisenberg's principle; in this sense, our state is the closest quantum analog to Popper's original proposal with a fixed and well localized source.

Note also that we have similarly chosen a Gaussian packet with $\Delta|2k_{1,2}| \simeq \Delta(k_1 - k_2) = \sigma_-$ to describe the vertical spread of the final momenta. Admittedly, this somehow reduces the generality of our treatment, but our Gaussian choice simplifies the analysis and by no means precludes the discussion of Popper's proposal which was intended to be valid for a generic wave packet. Physically, the momentum distribution is isotropic in s-wave decays, such as positronium annihilation into two photons; restricting to particles moving in the xy-plane, the vertical components of their momenta, $k_{1,2}$, are uniformly distributed in the range $-|\mathbf{k}_{1,2}| \le k_{1,2} \le +|\mathbf{k}_{1,2}|$. For vertically polarized spin-1 states decaying into two spinless particles, as in vector-meson decays into two pseudoscalar mesons, one has $k_{1,2} = |\mathbf{k}_{1,2}|\cos\theta$, where θ is the angle between $\mathbf{k}_{1,2}$ and the x-axis, and a vertical momentum distribution peaked around $k_{1,2} = 0$. Initial sources of higher spin can lead to distributions with more pronounced peaks around $k_{1,2} = 0$. We can somehow mimic this various possibilities by a judicious choice of σ_- in our Gaussian packet. Note finally that one has $\sigma_- \gg \sigma_+$ for any realistic value of $|\mathbf{k}_{1,2}|$.

3 Popper's proposal: case ii)

In order to discuss the standard quantum mechanical prediction for Δk_2 when the right-side slit is wide open and the width $2a$ of the left-side screen is modified, we need to Fourier transform state (2) into

$$\begin{aligned}\Psi(y_1;k_2;t) &= \int \frac{dk_1}{\sqrt{2\pi}} e^{ik_1 y_1} \\ &\quad\times \frac{1}{\sqrt{\pi\sigma_+\sigma_-}} e^{-\frac{1}{4}\left(\frac{1}{\sigma_+(t)^2}+\frac{1}{\sigma_-(t)^2}\right)(k_1^2+k_2^2)} e^{-\frac{1}{4}\left(\frac{1}{\sigma_+(t)^2}-\frac{1}{\sigma_-(t)^2}\right)2k_1k_2} \\ &= \frac{\sqrt{2}}{\sqrt{\pi\sigma_+\sigma_-}} \frac{\sigma_+(t)\sigma_-(t)}{\sqrt{\sigma_+(t)^2+\sigma_-(t)^2}} \\ &\quad\times e^{-\frac{1}{\sigma_+(t)^2+\sigma_-(t)^2}\left(\sigma_+(t)^2\sigma_-(t)^2y_1^2+k_2^2-i(\sigma_+(t)^2-\sigma_-(t)^2)y_1k_2\right)} .\end{aligned} \tag{3}$$

From these expressions it is easy to compute the probability for observing the vertical position of the left-moving particle within the range $-a \le y_1 \le +a$ allowed by the slit in coincidence with a given value, k_2, for the vertical component of the momentum of its right-moving partner. The former measurement requires detecting the left particle behind the slit using, for instance, a single detector placed on the negative x-axis far left of the slit. The measurement of the vertical component, k_2, of the right-side momentum is achieved thanks to the distant set of right-side detectors. The quantum mechanical prediction for the spread of the k_2 distribution in these coincidence measurements is then unambiguous:

$$(\Delta k_2)^2|_a = \frac{\int dk_2 k_2^2 |\int_{-a}^{+a} dy_1 \Psi(y_1;k_2;t)|^2}{\int dk_2 |\int_{-a}^{+a} dy_1 \Psi(y_1;k_2;t)|^2} . \tag{4}$$

We can now consider several values of the left-slit width $2a$. If this is infinitely narrow, $2a \to 0$ and $y_1 = 0$, one easily finds

$$(\Delta k_2)^2|_{a\to 0} = \frac{\sigma_+^2+\sigma_-^2}{4} \frac{1+\left(\frac{2\sigma_+\sigma_-}{\sigma_+^2+\sigma_-^2}\right)^2 \sigma_+^2\sigma_-^2\frac{t^2}{m^2}}{1+\sigma_+^2\sigma_-^2\frac{t^2}{m^2}} , \tag{5}$$

which decreases from $\frac{1}{4}(\sigma_+^2+\sigma_-^2)$ at $t=0$ to $\frac{\sigma_+^2\sigma_-^2}{\sigma_+^2+\sigma_-^2}$ when $t\to\infty$. Note that these results hold not only for $y_1 = 0$ but also for any other precise localization ($2a \to 0$) at a given y_1 of the left-moving particle.

We next increase the width of the left-side slit to a value $2a$ small enough to allow for an expansion of the y_1-Gaussian. Retaining the first three terms of the expansion, the quantum mechanical prediction for the k_2 distribution turns out to be

$$(\Delta k_2)^2|_a = (\Delta k_2)^2|_{a\to 0}(1-2a^2\delta) , \tag{6}$$

where

$$\delta = \frac{1}{12}\frac{(\sigma_+^2-\sigma_-^2)^2}{\sigma_+^2+\sigma_-^2}\frac{1}{1+\sigma_+^2\sigma_-^2\frac{t^2}{m^2}}\frac{1-\left(\frac{2\sigma_+\sigma_-}{\sigma_+^2+\sigma_-^2}\right)^2\sigma_+^2\sigma_-^2\frac{t^2}{m^2}}{1+\left(\frac{2\sigma_+\sigma_-}{\sigma_+^2+\sigma_-^2}\right)^2\sigma_+^2\sigma_-^2\frac{t^2}{m^2}} \ , \tag{7}$$

which is positive for reasonable values of t and thus $(\Delta k_2)^2|_a$ decreases when increasing the slit-width $s_L = 2a$.

We finally consider the other extreme case $a \to \infty$. From Eq. (4) one obtains

$$(\Delta k_2)^2|_{a\to\infty} = \frac{\sigma_+^2\sigma_-^2}{\sigma_+^2+\sigma_-^2} \ , \tag{8}$$

which is easily seen to be never larger than all the preceding results, Eqs. (5) and (6).

The predictions quoted in the last three paragraphs fully confirm the quantum mechanical analyses by Popper [3]–[5] on the dependence of Δk_2 on the left-side slit width $2a$. The larger (narrower) this width is chosen, the smaller (wider) is the k_2-dispersion Δk_2. It is easy to see that our treatment allows to confirm a related analysis by Peres [8] where the single left-side slit is substituted by a double slit thus producing interference effects on the right-side in coincidence measurements.

4 Popper's proposal: case i)

Let us finally move to the other possibility considered by Popper. In this case, one has to fix the width of left-side slit to $s_L = 2a$ and compare the predictions for Δk_2^2 when the right-side slit has the same width, $s_R = s_L = 2a$, with those from another setup with a wide open right-slit, $s_R \gg s_L = 2a$. This requires to perform a second Fourier transform of the state we are dealing with. It then reads,

$$\Psi(y_1,y_2;t) = \frac{\sigma_+(t)\sigma_-(t)}{\sqrt{\pi\sigma_+\sigma_-}}e^{-\frac{1}{4}\left[(\sigma_+(t)^2+\sigma_-(t)^2)(y_1^2+y_2^2)+(\sigma_+(t)^2-\sigma_-(t)^2)2y_1y_2\right]} \ . \tag{9}$$

From this expression it is easy to compute the spreading of the vertical position of the right-moving particle, $\Delta y_2^2|_a$, when it crosses the vertical plane $x = +d$ at time t. Simultaneously, its left-side partner passes through the window $-a \le y_1 \le +a$ allowed by the left-slit and will be detected much later. For these left-right coincidence detections, one has

$$(\Delta y_2)^2|_a = \frac{\int dy_2 y_2^2 |\int_{-a}^{+a} dy_1 \Psi(y_1,y_2;t)|^2}{\int dy_2 |\int_{-a}^{+a} dy_1 \Psi(y_1,y_2;t)|^2} \ . \tag{10}$$

For $a \to 0$ one obtains

$$(\Delta y_1)^2|_{a\to 0} \to 0 \ , \tag{11}$$

$$(\Delta y_2)^2|_{a\to 0} = \frac{1}{\sigma_+^2+\sigma_-^2}\frac{\left(1+\sigma_+^4\frac{t^2}{m^2}\right)\left(1+\sigma_-^4\frac{t^2}{m^2}\right)}{1+\sigma_+^2\sigma_-^2\frac{t^2}{m^2}} \geq 0 \ , \tag{12}$$

with $\Delta y_2^2|_{a\to 0} = \Delta y_1^2|_{a\to 0} = 0$ only if $t \to 0$ and $\sigma_\pm \to \infty$, i.e., in the case considered by Popper of a perfectly localized source ($\sigma_+ \to \infty$) and ignoring the wave packet spreading with t.

For finite but small a one similarly finds

$$(\Delta y_2)^2|_a = (\Delta y_2)^2|_{a\to 0}(1+2a^2\delta') \ , \tag{13}$$

where

$$\delta' = \frac{\sigma_+^2+\sigma_-^2}{12}\left[2\frac{1+\sigma_+^2\sigma_-^2\frac{t^2}{m^2}}{\left(1+\sigma_+^4\frac{t^2}{m^2}\right)\left(1+\sigma_-^4\frac{t^2}{m^2}\right)} - \frac{1+\left(\frac{2\sigma_+\sigma_-}{\sigma_+^2+\sigma_-^2}\right)^2}{1+\sigma_+^2\sigma_-^2\frac{t^2}{m^2}}\right] \ , \tag{14}$$

is never negative.

According to the results of the two last paragraphs, the spreading of the vertical right-side momentum, Δk_2, in coincidence events with a left-side particle passing through the $s_L = 2a$ wide slit, is indeed affected by the physical presence of an equal $2a$-wide slit on the right side. Its presence filters a narrower (in y_2) right-moving wave packet and this translates into a Δk_2 which is larger than in the case of removing (or making $s_R \gg 2a$ for) the right-side slit. This conclusion is in agreement with the analysis made by Short [10] of the optical experiment performed by Kim and Shih [9].

5 Conclusions

We have reconsidered Popper's test using the standard quantum mechanical formalism and, consequently, using a wave packet for the source —or, equivalently, for the initial two-particle system— which satisfies Heisenberg's principle, $\Delta(k_1+k_2)\Delta\left[(y_1+y_2)/2\right] \geq 1/2$. This contrasts with the original Popper's proposal involving a fixed source and therefore subjected to the criticisms raised by several authors [6, 7, 8, 10, 11, 12]. In spite of this and contrary to the claims of some of these authors [6]–[12], we find that Popper's test can be conclusive in that a narrowing of the left-side slit increases Δk_2^2 of the freely right-moving particle in coincidence detections. In other words, the qualitative behavior of Δk_2^2 that Popper attributes to standard quantum mechanics remains valid with our improved treatment of the initial state. In agreement with a related analysis by Short [10], we find however that the other aspect of Popper's proposal gets modified by our analysis; namely, Δk_2^2 necessarily increases when a second slit is *really* and symmetrically inserted on the right-side of the setup.

Our analysis shows that, to some extend, Popper's test can indeed be conclusive to discriminate between his own approach and the standard version of quantum mechanics. The latter turns out to be favored by recent optical experiments, which are somehow related to the original proposal [9, 13, 14, 15]. Quantum non-locality, a key question in all these discussions, is nowadays firmly established. Recent experiments tend to falsify Popper's approach but his understanding of quantum mechanics as early as in 1934 [2] is quite remarkable.

Note added: After publication of the present paper in the Proceedings of the Fundamental Physics Meeting "Alberto Galindo" we have received an improved version of Qureshi's paper in Ref. [12]. The state discussed in this new version coincides with our Eqs. (1) and (2) once we define $\sigma_+^2 = 1/4\Omega_0^2$ and $\sigma_-^2 = 4\sigma^2$.

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References

1. Galindo A, Pascual P (1978) Mecánica Cuántica; ed. Alhambra, Madrid
2. Popper K R (1934) Die Naturwissenschaften 22:807
3. Popper K R (1982) Quantum Theory and the Schism in Physics. Hutchinson, London
4. Popper K R (1984) Open Questions in Quantum Physics. Tarozzi G, van der Merwe A (eds) Dreidel, Dordrecht
5. Popper K R (1987) Nature 328:675 and (E) Nature 329:112
6. Bedford D, Selleri F (1985) Lett Nuov Cim 42:325
7. Collett M J, Loudon R (1987) Nature 326:671 and Nature 328:675
8. Peres A (2002) Stud History Philos Modern Physics 33:23
9. Kim Y H, Shih Y H (1999) Found Phys 29:1849
10. Short A K (2001) Found Phys Lett 14:275
11. Qureshi T (2004) Int J Quant Inf 2:1
12. Qureshi T (2004) quant-ph/0405057
13. Strekalov D V (1995) Phys Rev Lett 74:3600
14. Zeilinger A (1999) Rev Mod Phys 71:S288
15. Dopfer B (1998) Thesis, University of Innsbruck